5.6 Transforming Quadratics

Transforming quadratic functions works just like transforming any other function. However, to be a bit more specific with quadratics, we'll spend more time looking at vertex form. First, a quick review of the general function notation transformations.

Transforming $f(x)$

In general, we have talked about translating functions up or down, translating functions left or right, and stretching functions away from (or closer to) the axes. Here's the quick review:

Translate up/down

$f(x) + c$

If c is positive, the function shifts up.

If c is negative, the function shifts down.

Translate left/right

$f(x + c)$

If c is positive, the function shifts left.

If c is negative, the function shifts right.

Farther from x-axis

$c(fx)$

If $c > 1$, the function goes farther from x-axis.

If $0 < c < 1$, the function goes closer to x-axis.

If c is negative, the function reflects across x-axis.

Closer to y-axis

$f(cx)$

If $c > 1$, the function goes closer to y-axis.

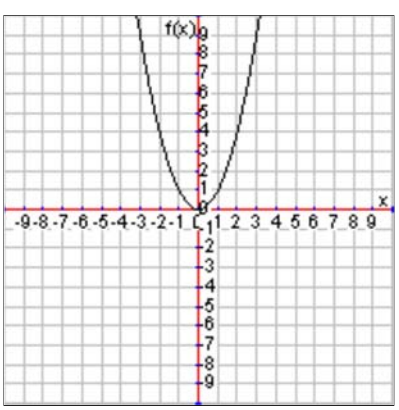
If $0 < c < 1$, the function goes farther from y-axis.

If c is negative, the function reflects across y-axis.

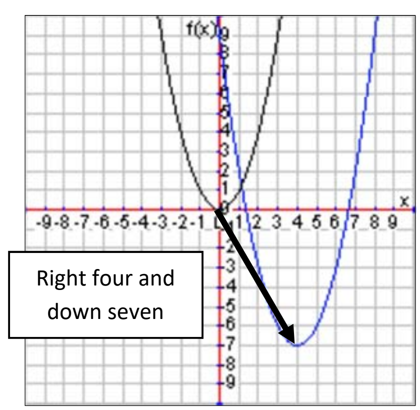
Transforming $f(x) = a(x - h)^2 + k$

Let's talk specifically about transforming quadratics in vertex form. Since the point (h, k) is the vertex, those values give us our shift left/right and up/down. Consider the parent function $f(x) = x^2$. Then the function $g(x) = (x - 4)^2 - 7$ has been translated four to the right and seven down because the vertex is at $(4, -7)$.

Parent Function $f(x) = x^2$

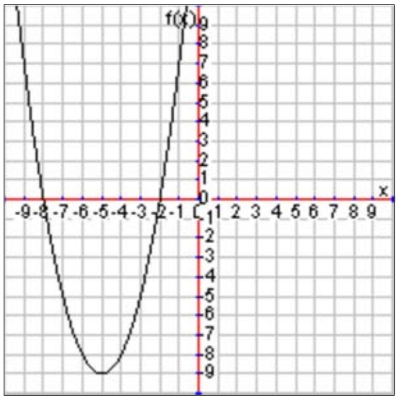


Transformed Function $g(x) = (x - 4)^2 - 7$

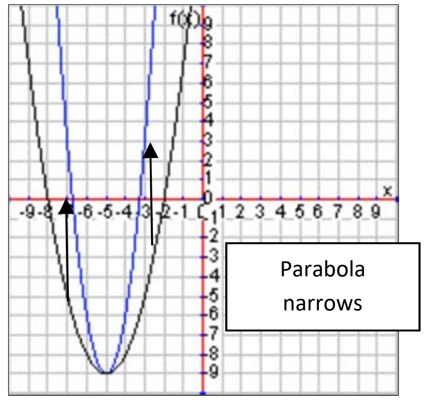


The a value in vertex form is a bit different. Sometimes that is called the stretch value. Let's think about why. If $a > 1$, then the transformed function output will be higher than the parent function. Since a parabola is "U" shaped, this will mean the parabola will narrow as it goes up faster on both sides of the vertex.

Original Function $f(x) = (x + 5)^2 - 9$

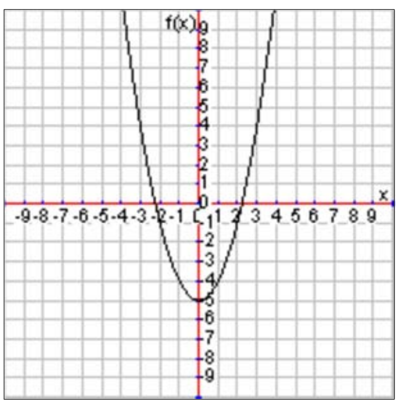


Transformed Function $g(x) = 3(x + 5)^2 - 9$

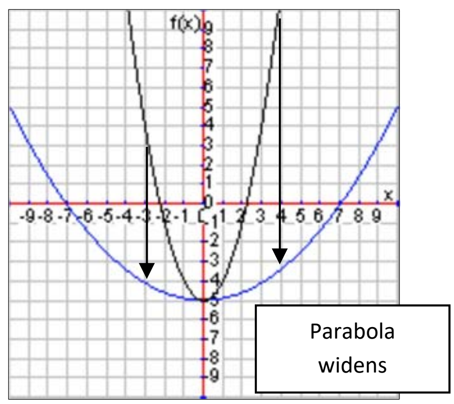


If $0 < a < 1$, then the transformed function output will be smaller than the parent function. Since a parabola is "U" shaped, this will mean the parabola gets wider as it goes up slower on both sides of the vertex.

Original Function $f(x) = x^2 - 5$

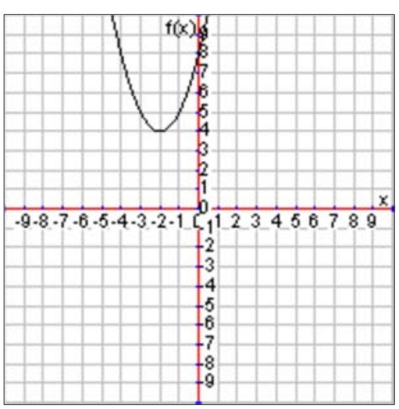


Transformed Function $g(x) = \frac{1}{10}x^2 - 5$

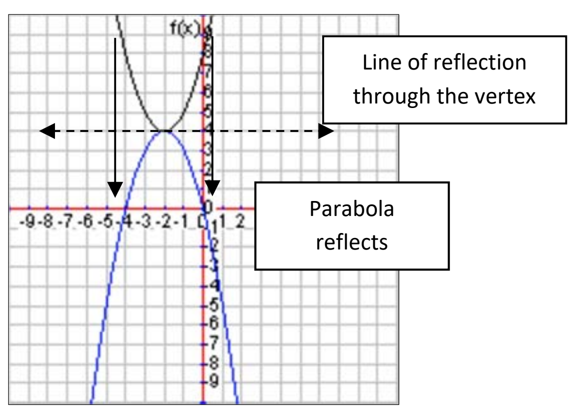


If a is negative, then the transformed function output will be reflected across the horizontal line through the vertex. In other words, it will flip upside down.

Original Function $f(x) = (x + 2)^2 + 4$



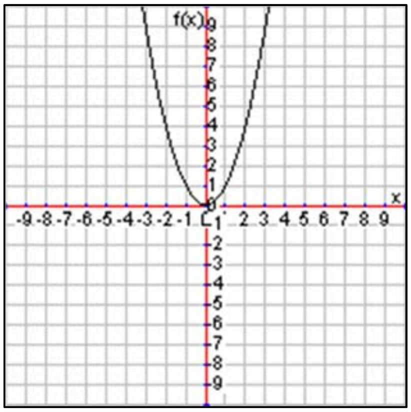
Transformed Function $g(x) = -(x + 2)^2 + 4$



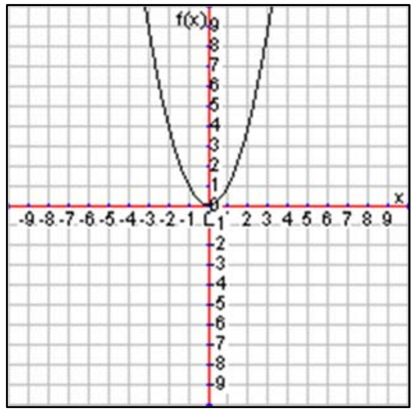
Lesson 5.6

Describe the transformation applied to the parent function $f(x) = x^2$. Then do a quick sketch of each one.

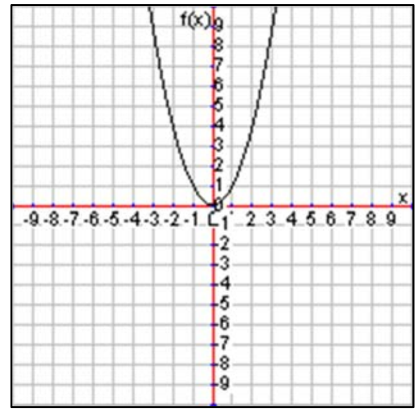
1. $f(3x) + 2$



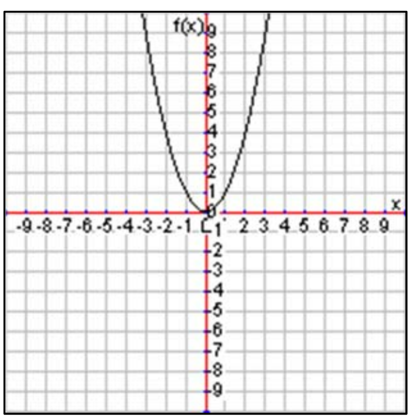
2. $\frac{1}{2} * f(x - 2)$



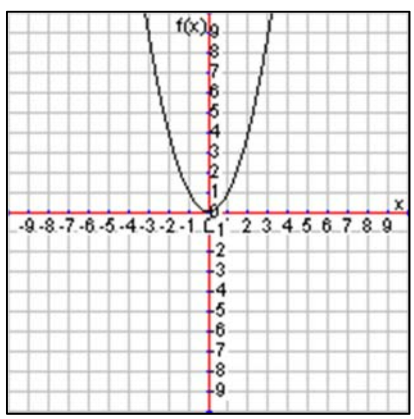
3. $-f(x + 5)$



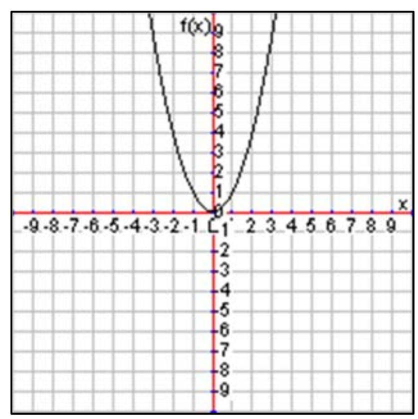
4. $g(x) = (x - 1)^2$



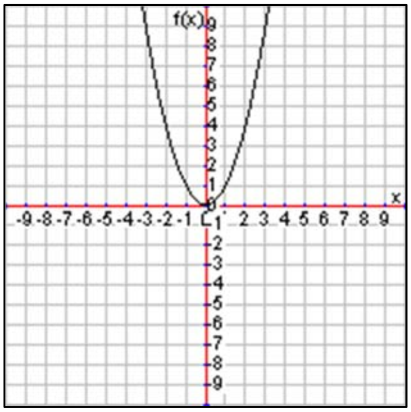
5. $g(x) = 2x^2 - 8$



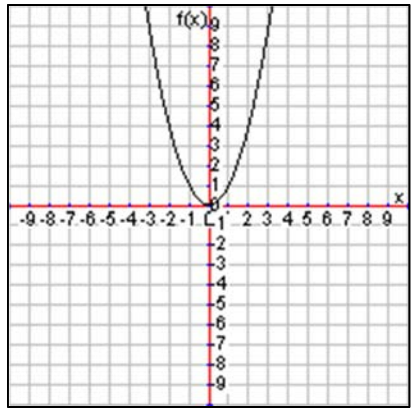
6. $g(x) = -(x + 6)^2$



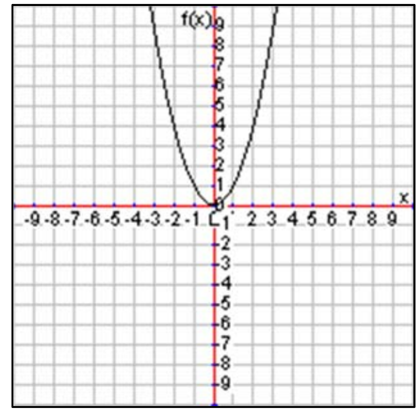
7. $g(x) = \frac{1}{3}x^2 + 3$



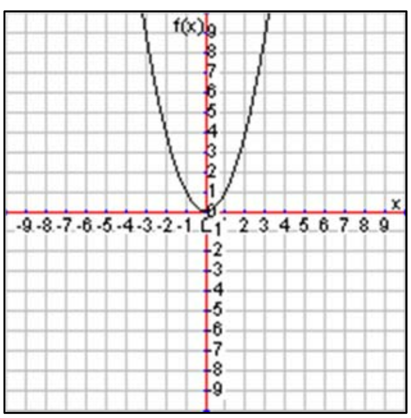
8. $g(x) = (x + 2)^2 - 5$



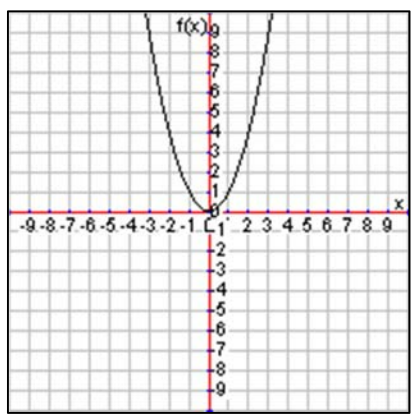
9. $g(x) = 2(x - 3)^2$



10. $g(x) = -x^2 - 8x - 16$



11. $g(x) = x^2 - 2x - 5$



12. $g(x) = 2x^2 - 12x + 18$