Lesson 5: Transformations of Exponential Functions

Part A – Introduction

Recall:

- Any function $y = f(x)$ can be transformed according to $y = af[k(x-d)]+c$. Therefore all the transformations can be applied to the exponential function as follows: $y = ab^{k(x-d)} + c$.

- Always factor first, if needed;
    - ***a*** determines vertical stretch or compression
    - ***k*** determines the horizontal stretch /compression
    - ***d*** determines the horizontal translation left or right
    - ***c*** determines the vertical translation up or down

- Apply the transformations in this order (i) horizontal/vertical stretches
  (ii) reflections
  (iii) translations

- Mapping technique: $(x, y) \to \left(\frac{x}{k} + d, ay + c\right)$

Part B – Anatomy of the Transformed Exponential Function

Let $f(x) = 2^x$, applying all the transformations we get,

$$g(x) = -4f(-3x-9)+6$$

$$= -4(2)^{-3(x+3)} + 6$$

Part C – Examples

Graph each of the following functions. State the y-intercept, the equation of the asymptote, domain and range.

a) $y = -2^{x} + 4$ (Hint: Graph the new asymptote first; it's given by $y = c$)

y-intercept: ________________ equation of asymptote: ______________

domain: ___________________ range: ________________________

b) $y = 3^{-(x-4)}$

y-intercept: ________________ equation of asymptote: ______________

domain: ___________________ range: ________________________

c) $y = \left(\frac{1}{2}\right)^{3x} - 2$

y-intercept: ________________ equation of asymptote: ______________

domain: ____________________ range: _________________________

Worksheet:

Graphing Transformations of $y = b^x$.

Graph each of the following exponential functions, and determine:

(i) the equation of the horizontal asymptote
(ii) the y-intercept
(iii) the domain and range

a) $y = 2(3^x - 1)$

b) $y = \frac{1}{3}\left(2^{(x+4)}\right)$

c) $y = -(3^x) + 2$

d) $y = 2\left(\frac{2}{3}\right)^{-x} - 3$