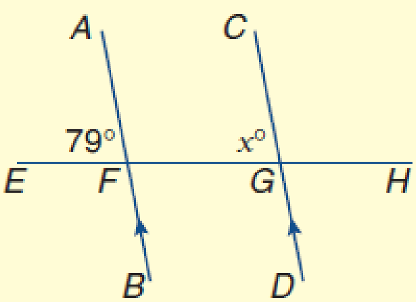
1:04 | Geometry

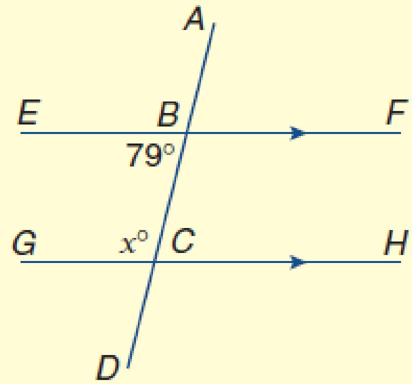
Exercise 1:04

1 **a**



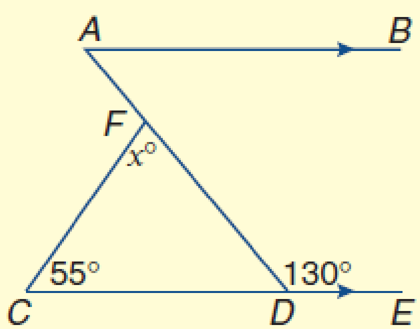
Find x. Give reasons.

b



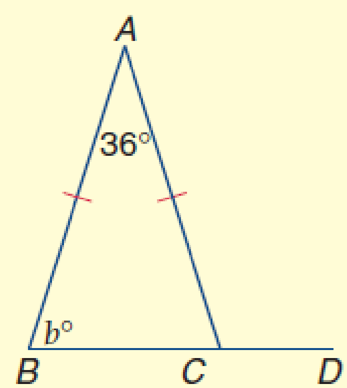
Find the size of x. Give reasons.

c



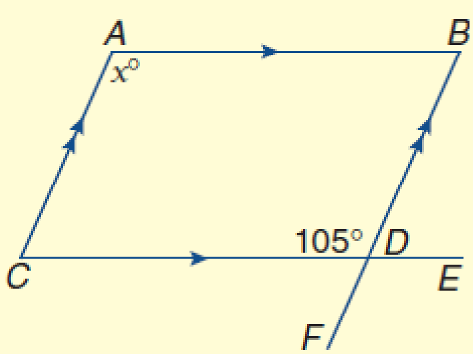
Find the size of x.
Give reasons.

d



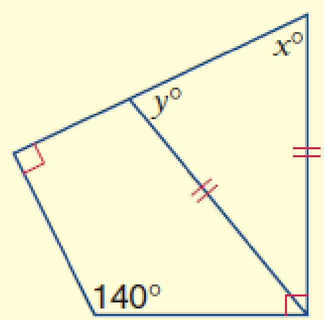
Find the value of b.
Give reasons.

e



ABDC is a parallelogram. Find the size of x. Give reasons.

f



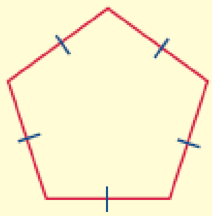
Find the value of x and y.
Give reasons.

2 **a** What is the sum of the interior angles of:

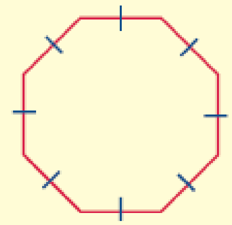
i a hexagon? **ii** a decagon?

b What is the size of each interior angle in these regular polygons?

i



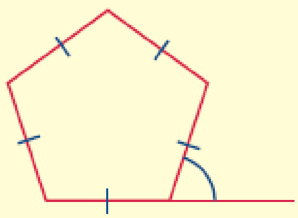
ii



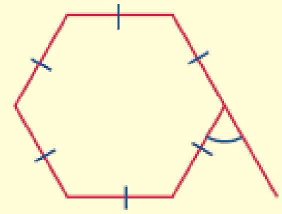
c What is the sum of the exterior angles of an octagon?

d Find the size of each exterior angle of these regular polygons.

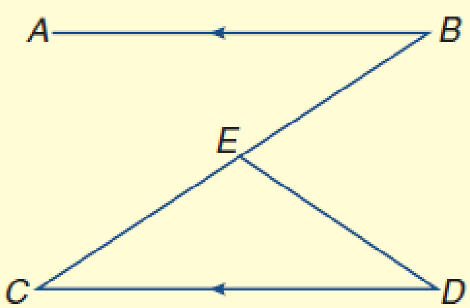
i



ii

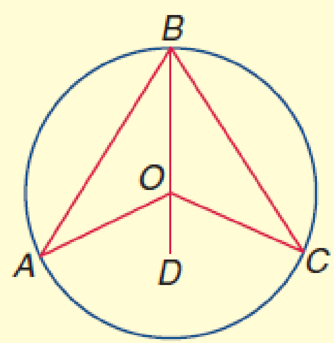


3 **a**



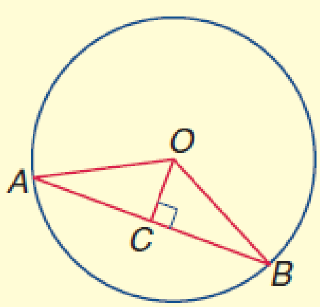
Prove $\angle BED = \angle ABC + \angle CDE$.

b



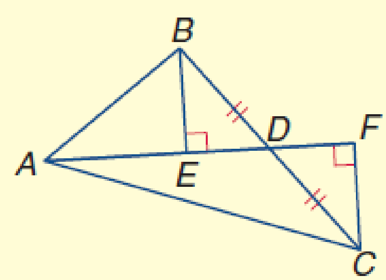
O is the centre of the circle.
Prove that $\angle AOC = 2 \times \angle ABC$.
(*Hint:* $AO = BO = CO$ (radii).)

4 **a**



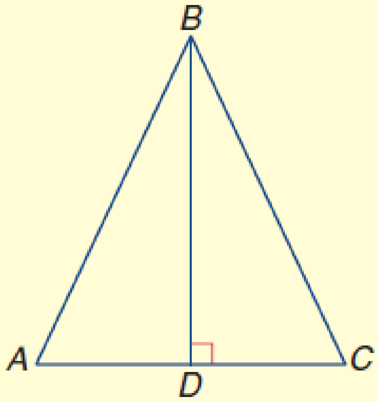
O is the centre and $OC \perp AB$.
Prove that $\Delta OCA \equiv \Delta OBC$ and hence that $AC = BC$.

b



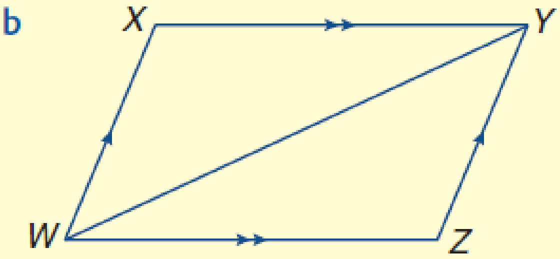
ΔABC is any triangle. D is the midpoint of BC, and BE and CF are perpendiculars drawn to AD, produced if necessary.
Prove that $\Delta BED \equiv \Delta CFD$ and hence that $BE = CF$.

5 a



In ΔABC, a perpendicular drawn from B to AC bisects $\angle ABC$. Prove that ΔABC is isosceles.

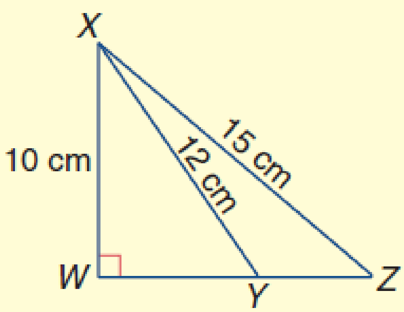
b



$WXYZ$ is a parallelogram, ie $WX \parallel ZY$ and $WZ \parallel XY$. Prove $\angle WXY = \angle YZW$

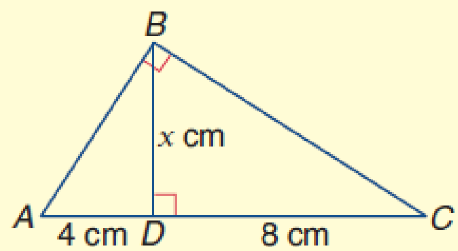
(*Hint:* Use congruent triangles.)

6 a



Find the value of YZ.

b



i Find an expression for AB.

ii Find an expression for BC.

iii Hence, find the value of x.

Answers:

Exercise 1:04

1 **a** $x = 79$ (corresp. $\angle$s, $\parallel$ lines)

b $x + 76 = 180$ (co-int. $\angle$s, $\parallel$ lines)
$x = 104$

c $x + 55 = 130$ (ext. $\angle$ of Δ)
$x = 75$

d $\angle ACB = b°$ (base $\angle$s of isos. Δ)
$b + b + 36 = 180$ ($\angle$ sum of Δ)
$\therefore b = 72$

e $x = 105$ (opp. $\angle$s of a par'm)

f $x = 40$ ($\angle$ sum of a quad.)
$y = 40$ (base $\angle$s of isos. Δ)

2 **a** **i** 720 **ii** 1440 **b** **i** 108° **ii** 135° **c** 360° **d** **i** 72° **ii** 60°

3 **a** $\angle BED = \angle ECD + \angle CDE$ (ext. $\angle$ of Δ)
$\angle ABC = \angle ECD$ (alt. $\angle$s, $AB \parallel CD$)
$\therefore \angle BED = \angle ABC + \angle CDE$

b ΔAOB is isosceles
since $AO = BO$ (radii)
$\therefore \angle OAB = \angle OBA$ (base $\angle$s isos. Δ)
$\angle DOA = \angle OAB + \angle OBA$ (ext. $\angle$ of Δ)
$\therefore \angle DOA = 2 \times \angle OBA$
Similarly, $\angle DOC = 2 \times \angle OBC$
$\therefore \angle DOA + \angle DOC = 2 \times (\angle OBA + \angle OBC)$
ie $\angle AOC = 2 \times \angle ABC$

4 **a** In Δs OCA and OBC
OC is common
$AO = BO$ (radii)
$\angle OCA = \angle OCB = 90°$ (given)
$\therefore \Delta OCA \equiv \Delta OBC$ (RHS)
$\therefore AC = BC$ (corres. sides in cong. Δs)

b In Δs BED and CFD
$BD = CD$ (D is midpt BC)
$\angle BED = \angle CFD = 90°$ (given)
$\angle BDE = \angle CDF$ (vert. opp. $\angle$s)
$\therefore \Delta BED \equiv \Delta CFD$ (AAS)
$\therefore BE = CF$ (corres. sides in cong. Δs)

5 **a** In Δs ABD and CBD
BD is common
$\angle BDA = \angle BDC = 90°$ ($BD \perp AC$)
$\angle ABD = \angle CBD$ (AC bisects $\angle ABC$)
$\therefore \Delta ABD \equiv \Delta CBD$ (AAS)
$\therefore AB = CB$ (corres. sides in cong. Δs)
$\therefore \Delta ABC$ is isos. (2 equal sides)

b In Δs WXY and YZW
WY is common
$\angle XWY = \angle ZYW$ (alt. $\angle$s, $WX \parallel YZ$)
$\angle XYW = \angle ZWY$ (alt. $\angle$s, $XY \parallel WZ$))
$\therefore \Delta WXY \equiv \Delta YZW$ (AAS)
$\therefore \angle WXY = \angle YZW$ (corres. $\angle$s in cong. Δs)

6 **a** $YZ = 4{\cdot}55$ cm (to 2 dec. pl.)

b **i** $AB^2 = x^2 + 4^2$
ii $BC^2 = x^2 + 8^2$
iii ie $AB^2 + BC^2 = 2x^2 + 16 + 64$
$= 2x^2 + 80$
But $AB^2 + BC^2 = 12^2 = 144$
So $2x^2 + 80 = 12^2 = 144$
ie $x = \sqrt{32}$ ($\doteqdot 5{\cdot}6$)

Exercise 1:05